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FRANK LUDVIG SPITZER

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A Biographical Memoir by
HARRY KESTEN

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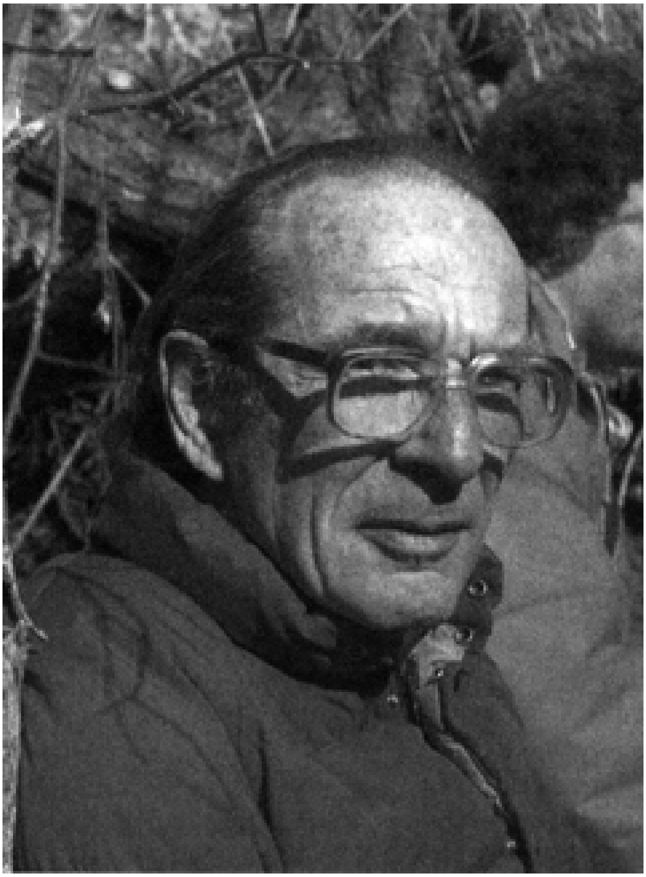

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FRANK LUDVIG SPITZER

July 24, 1926–February 1, 1992

BY HARRY KESTEN

FRANK SPITZER WAS A highly original probabilist and a humorous, charismatic person, who had warm relations with students and colleagues. Much of his earlier work dealt with the topics of random walk and Brownian motion, which are quite familiar to probabilists. Spitzer invented or developed quite new aspects of these, such as fluctuation theory and potential theory of random walk (more about these later); however, his most influential work is undoubtedly the creation of a good part of the theory of interacting particle systems. Through the many elegant models that Frank constructed and intriguing phenomena he demonstrated, a whole new set of questions was raised. These have attracted and stimulated a large number of young probabilists and have made interacting particle systems one of the most exciting and active subfields of probability today.

Frank Spitzer was born in Vienna, Austria, on July 24, 1926, into a Jewish family. His father was a lawyer. When Frank was about twelve years old his parents sent him to a summer camp for Jewish children in Sweden. Quite possibly the intention of this camp was to bring out Jewish children from Nazi-held or Nazi-threatened territory. Be that as it may, Frank's parents soon informed him that the situation was too precarious in Austria for him to return, and consequently Frank spent World War II in Sweden. He lived there in succession with two Swedish families, learned Swedish, and went through high school. He also attended Tekniska Hogskolan in Stockholm for one year. Somehow during the war his parents and sister made their way to the United States through the unoccupied part of France and North Africa. After the war Frank followed them to the United States, where he soon entered the army. In 1947 after his military service he entered the University of Michigan in Ann Arbor. In part because Frank managed to talk the University of Michigan into giving him college credit for several of his high school courses in Sweden, he completed his B.A. and Ph.D. in Michigan in a mere six years (1947-53). Part of this time he was actually away from Michigan. One of his leaves was for an extended visit to Princeton, where he met the famous probabilist William Feller.

For financial support Frank drove a cab for a while in Ann Arbor. He also met and married his first wife, Jean Wallach, in Ann Arbor. Jean and Frank had two children, a daughter Karen and a son Timothy. In the mid-seventies this marriage ended in divorce, and Frank started a second marriage with Ingeborg Wald. Frank is survived by both his partners and his two children.

Two classical stochastic processes are random walk and Brownian motion. Random walk is often used as a model to describe an evolving quantity which can be observed, or which is meaningful, only for a discrete sequence of times. The value of such a process at the k-th observation is then typically denoted by S_k, with k running through the integers, or sometimes only through the positive integers. $X_{k+1} := S_{k+1} - S_k$ is then the increment of the process from the k-th to the $(k + 1)$-th observation and $S_k = S_0 + \sum_1^k X_i$. In a random walk the increments X_k are assumed to be independent and identically distributed. Roughly speaking, this means that all X_k for different k have the same statistical properties, and that the value of any X_k has no influence on the values of the X_j for $j \neq k$. A traditional example which can be modeled this way is a gambling situation in which one repeatedly plays the same game; X_k represents the gain of a given player during the k-th game. Brownian motion (also called a Wiener process because Norbert Wiener was the first to give a rigorous construction of this process) is used in some situations in which it is more appropriate to build a model with time varying continuously from $-\infty$ to ∞ or from 0 to ∞, rather than having time restricted to a discrete sequence. A Brownian motion $\{B(t)\}_{t\geq 0}$ has some fundamental similarities with random walk. For disjoint time intervals $[t_1,t_2]$ and $[s_1,s_2]$, the increments $B(t_2) - B(t_1)$ and $B(s_2) - B(s_1)$ are independent, and when the intervals have the same length ($t_2 - t_1 = s_2 - s_1$), then these increments even are identically distributed. In fact these increments all have a Gaussian (also called normal) distribution. In the simplest case the Brownian motion is one-dimensional, that is $B(t)$ is a real number. However, one also considers d-dimensional Brownian motion in which $B(t)$ is a d-dimensional vector. Brownian motion has many fascinating properties. For instance, its paths are continuous but nowhere differentiable; for a while in the nineteenth century mathematicians had even doubted that such functions exist.

Random walks as well as Brownian motion figured prominently in Frank's research.[1] My impression is that he usually picked his own research problems and that little of his work is due to direct guidance or influence of other probabilists; however, from some of his remarks I gather that the contacts with Feller, whom Frank met at Princeton, did have an important influence on his thesis. In the preface of his book (1964,1) Frank thanks "those of my teachers, Donald

Darling, William Feller, and Samuel Karlin, who introduced me to the theory of stochastic processes." His thesis was on two-dimensional Brownian motion $\{B(t)\}$ (that means that $B(t)$ takes values in the plane). It had been known for some time[2] that a two-dimensional Brownian motion does not visit the origin in the plane, but does come arbitrarily close to the origin. In his thesis and a 1958 paper based on it, Spitzer estimated how close a two-dimensional Brownian motion comes to the origin during a long time interval. Another peculiarity of two-dimensional Brownian motion is that it winds around the origin an arbitrarily large number of times. But it also unwinds itself and winds in the other direction infinitely often during its history. Spitzer further found the distribution of the winding number of the Brownian motion at a given time. This has led to many further investigations of the joint distributions of winding numbers with respect to more than one point. Yor[3] gives an impression of how far these investigations have gone. In (1964,2) Spitzer returned to Brownian motion and gave a limit theorem for the volume of the so-called Wiener sausage, the volume swept out by a ball whose center undergoes a Brownian motion (in dimension $d \geq 3$).

Frank's first academic position was at the California Institute of Technology as instructor from 1953 to 1955 and as assistant professor from 1955 to 1958. While there he became acquainted with Sparre Andersen's remarkable papers[4] which dealt with the maximum,

$$M_n = \max_{0 \leq k \leq n} S_k,$$

of a random walk $\{S_k\}$ and the time at which this maximum is attained. Sparre Andersen showed that several relations held for these quantities independently of the distribution of the increments $\{X_i\}$. These properties derive entirely from the fact that $X_1, X_2, \ldots, X_n$ has the same distribution as any rearrangement $X_{\sigma(1)}, X_{\sigma(2)}, \ldots, X_{\sigma(n)}$ of this sequence (σ here is a permutation of $\{1,2,\ldots,n\}$). These results came as a considerable surprise to the probability community at that time, because limit relations for M_n so far had been based on specific assumptions on the distribution of the X_i. Spitzer realized what the basic combinatorial principles behind Sparre Andersen's results were and he greatly extended Sparre Andersen's papers. For instance, he showed that for any sequence $\{x_1, x_2, \ldots, x_n\}$, the values taken on by the maximum

$$m_n(\sigma x) = \max_{0 \le k \le n} \sum_{j=1}^{k} x_{\sigma(j)}$$

are the same as the values of

$$T(\tau x) = \sum_i \left(\sum_{j \in i-th \text{ cycle of } \tau} x_{\tau(j)} \right)^+$$

as σ and τ both run over the $n!$ permutations of $\{1,2,\ldots,n\}$. (Spitzer [1956] credits Bohnenblust with help on this proof.) When the x_i are replaced by independent identically distributed random variables X_i, then the T is much easier to deal with than the maxima. This led to the celebrated expression in (1956) for the generating function

$$\sum_0^\infty E \exp[-\lambda M_n] t^n,$$

where E denotes the mathematical expectation or average. This result is now known as the Pollaczek-Spitzer formula; Pollaczek[5] had in fact earlier derived the same formula by a much more complicated route and under more restrictive conditions. The general area of Frank's (1956) paper is now known as fluctuation theory.

From the California Institute of Technology Frank moved in 1958 to the University of Minnesota. Many of the earlier limit theorems on maxima of random walk had been developed at Cornell (by Kac, Erdös, Chung, and Sparre Andersen) and it was natural for Frank to visit Cornell at some time. He did so during the summers of 1958 and 1960.[6] This led to a move in 1961 to Cornell as a full professor, and, with the exception of a number of sabbatical and study leaves, Frank stayed there for the rest of his life. For a number of years at Cornell Frank worked on the development of potential theory for random walk. Since the famous work of Kakutani[7] and Doob[8] it had been known that there is a close connection between classical potential theory and Brownian motion. For instance, Green's function in d dimensions has an immediate interpretation in terms of the expected amount of time a d-dimensional Brownian motion spends in subsets of d-space. This works well when $d \geq 3$, when the Brownian motion is transient (that is, stays outside any fixed ball eventually). Also, the distribution of the position where a Brownian motion first hits a set can be used to solve Dirichlet's problem. Hunt[9] extended this relationship to situations where the Brownian motion is replaced by any transient Markov process. Spitzer then asked what the analogous results were for random walk, and more importantly, what the analogous results were for a recurrent random walk. Such a random walk spends an infinite amount of time in any ball and one cannot simply use the expected amount of time spent in a subset as an analogue for Green's function, because this quantity is usually infinite. This led Spitzer to study the so-called recurrent potential kernel. For an integer valued random walk $\{S_n\}$ this is given by

$$a(x) = \sum_{n=0}^{\infty} [P\{S_n = x\} - P\{S_n = 0\}].$$

Frank's deepest theorem of those years is probably that this sum converges for any random walk on the integers, without conditions on the distribution of the increments. He further showed that this is indeed a "good" potential kernel in the sense that one can write the solution of certain equations in terms of this kernel, and he studied the asymptotic behavior of this kernel. In turn, this allowed him to obtain limit theorems for the distribution of the position where a random walk first hits a given set. As a measure of the difficulty of these results it should be pointed out that it is still not known whether the series of $a(x)$ always converges absolutely (Spitzer only showed conditional convergence). An excellent and readily accessible exposition of these results (and many more) can be found in Spitzer's elegant book (1964,1).

In a random walk $\{S_n\}$, S_n is sometimes interpreted as the position of a particle at time n. The assumption that the increments of the random walk are independent and identically distributed is reasonable when the particle moves entirely without influence of other particles. It is, however, a very simplifying assumption and is not justified in most statistical mechanics models. Even today it still is too difficult to analyze probability models which realistically deal with the interactions and collisions of molecules in a gas, say; however, Harris[10] had already considered some simplified models which incorporated collisions for particles which moved on the line. Perhaps stimulated by this, but in any case also by his desire to get away from the classical independence assumptions and to find new phenomena, Spitzer began in the late sixties to investigate a number of probabilistic models in which there are more interactions. In this vein he invented the "random walk in random environment" model. For a random walk in random environment the distribution of the increment X_{k+1} depends on the position at time k, S_k. This dependence is itself random. Formally, one first chooses a random environment which prescribes for each possible position x, what the distribution of X_{k+1} will be when $S_k = x$ (for some k). Once the environment is fixed the particle moves in this environment according to the transition rules specified by the environment. The model and its many later generalizations are of considerable interest and challenge to probabilists because of their non-Markovian nature; the full sequence of past observations gives us more information about the environment than just the last observations. In fact, by observing the successive positions of the random walk one finds out more and more about the environment even though the environment itself cannot be observed directly. During a visit to the Soviet Union in the early seventies Spitzer found that the same model had been independently invented there as a highly simplified and mathematicized model for DNA replication. This led to the joint paper (1975,4) with one author from the Soviet Union whom Frank met on his trip. Random walk in random environment is a model in which randomness is introduced in two stages, first in the choice of the environment (or equivalently, parameters for the transition mechanism), and then in the motion of the particle. Since 1975 probability models with such "two-stage" randomness have become very fashionable in probability as well as statistical physics.

More or less in the same period Spitzer also began to study models in which many (often infinitely many) particles interact locally. Nowadays we call these models "interacting particle systems." Closely related investigations were taking place in the Soviet Union by Dobrushin and his school.[11] Dobrushin's work was directly motivated by statistical physics, in particular by the Ising model for magnetism. Dobrushin was one of the people who gave a precise definition of Gibbs states (which generalize the Ising model) and who has contributed heavily to the study of their properties. It is not clear how much Frank had statistical mechanics in mind when he started looking at interacting particle systems, but it soon became an important factor. Various examples in interacting particle systems represent time evolutions which have well known statistical mechanics models for their equilibrium state. Like statistical mechanics models, several interacting particle systems exhibit a phase transition. In fact, it was precisely for such properties that Frank and Dobrushin selected some of their models for study. Because of this, interacting particle systems are responsible, in part, for the renewed interaction and cooperation between statistical physicists and probabilists taking place these days. Even though this is probably the area in which Frank had the greatest influence we have to restrict ourselves here to just two illustrations of models which he invented.[12] The first is the simple exclusion model which he introduced in (1970). Assume that there are infinitely many particles with positions in the integers. Each particle decides on its own (without influence from the other particles) when it would like to change position and where it would like to move. The interaction now comes from the single rule that at any time no two particles are allowed to occupy the same site. Thus, if a particle at position x decides at time t that it wants to move to site y, but y happens to be occupied at time t, then this move is suppressed and the particle at x stays there (until its next attempt to move somewhere). To complete the description of the model one must of course specify when and how a particle wants to jump. Often one assumes that these jumps follow a continuous time analogue of a random walk (or more generally a Markov process). In the exclusion model no particles are created or disappear. This is not the case for the second model, the so-called "nearest particle system." Again one looks at a system of particles on the integers. But now each particle that is present can disappear at a fixed death rate δ and also particles can be born at an unoccupied site x at a rate which is assumed to be a function of the distances to the nearest occupied sites on the left and right of x. The first question for these models is whether there exists a decent process which corresponds to the above description; the trouble is that in theory in the exclusion model infinitely many particles may try to jump to a given site in finite time. Similarly, in the nearest particle system an infinite chain of births and deaths could conceivably occur in a finite time interval. Frank did not do much work on this existence problem by himself, but it has now been adequately solved. (See Liggett's book mentioned in note 12 for an exposition of this. This book also has many results about the nearest particle system to which Liggett himself has made major contributions.)

The next questions considered by Frank for various interacting particle systems were what the equilibrium distributions are for such processes and whether the state of the process converges to such an equilibrium distribution from suitable initial states. In many examples there is an analogue of a phase transition; for some parameter values in the transition mechanism there is a unique equilibrium state and for others there are more than one. The unraveling of these "phase diagrams" and the description of the domains of attraction of the various equilibrium distributions has been a fundamental goal of the field. For a number of examples Frank found all or some equilibrium states (1970, 1977). Such explicit results pleased him greatly. In other cases he discovered a so-called duality relation which is a basic tool for proving convergence to equilibrium. The theory has become very rich and many other questions have arisen.

An important topic nowadays is to describe how the system approaches equilibrium from suitable initial states. By taking suitable scaling limits this can often be described by partial differential equations for a local particle density (hydrodynamic limits).[13] One also tries to estimate probabilities of large deviations from equilibrium behavior and questions of metastability;[14] that is, if the system has one equilibrium state but starts out "far away" from this equilibrium state, how long can it stay far away? Another direction is to introduce particles of different types and to investigate when there are equilibrium states in which several of the types can coexist.[15] This direction has been stimulated by biological interpretations. As mentioned before, interacting particle systems are a very active, exciting area, and Frank was one of the founding fathers.

Spitzer was elected to the National Academy of Sciences in 1981 and held a Guggenheim fellowship in 1965-66. He was invited for many prestigious lectures, including a lecture at the International Congress of Mathematicians at Vancouver in 1974 and the Wald Lectures for the Institute of Mathematical Statistics in 1979. For almost twenty years he was on the editorial board of one of the major probability journals, the *Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete* and its successor, *Probability Theory and Related Fields*. He also taught special courses at various summer schools.

Frank had great enthusiasm for his field. Not only did he love his own work, but, what is much rarer, he knew how to show genuine appreciation for the work of others. I often felt much encouraged by his comments, and from various messages that I received from colleagues and former students after his death, I know they felt the same. Any secretiveness about his ongoing research was totally alien to Frank. On the contrary, he usually tried to draw colleagues and students into cooperating with him. In this way I became his coauthor on a number of papers and even owe some papers by myself to Frank's questions and stimulation. He was quite generous with his time and on many occasions helped his students and teaching assistants with nonmathematical problems. Because of this and his spontaneity many people close to Frank felt great warmth towards him. Until he developed Parkinson's disease he was an inspired teacher. In fact, he even enjoyed teaching nonmathematical subjects. He taught me the rudiments of skiing and how to use the T bar and ski lift on the local ski slope. Frank knew how to help students over hurdles and once volunteered to give a pep talk to one of my Ph.D. students who seemed to have given up on his thesis work. Frank's talk had the desired effect and the student did finish his Ph.D. Of course, Frank also had considerable influence on his own Ph.D. students, and several of them have now made a career for themselves in interacting particle systems. A strong sense of elegance guided Frank in his research. For this reason he worked so hard to prove the convergence of the series for the potential kernel $a(x)$ mentioned before, without any conditions on the distribution of the increments. The great attraction of this result is that it has no extra conditions. Frank was always toying with probability models, looking for new phenomena. He did not particularly like extending the validity of some known results if this did not lead to some surprises. Once he complained about a visitor who had treated him to several blackboards full of formulae. "What did he want me to do? Eat a bunch of formulae?," Frank asked me. Probably because of these standards of his, he did not publish all that many papers (about 50), but he has helped shape probability theory as we know it today.

Frank had a great love for the outdoors, and, even though he never became a real expert, he was an avid mushroom hunter. He loved to ski. In fact, in his office he displayed with pride a list of times for a downhill run for a number of mathematicians during a "race" in which Frank had participated during his visit to the Soviet Union. He often went hiking, especially in the mountains, and cross-country skiing. He regularly went jogging almost until the end of his life, even after this became difficult because of his struggle with Parkinson's disease. In addition to Parkinson's disease Frank contracted bladder cancer. The immediate cause of his death was a urinary tract infection which seemed to have been related to the chemotherapy he was undergoing for his cancer.

I AM INDEBTED TO Jean Spitzer for a number of biographical data and to Thomas Liggett for some comments about Spitzer's work on interacting particle systems. I thank Geoffrey Grimmett for the photograph of Frank Spitzer and for several helpful suggestions for this memoir.

NOTES

1. Much of the following description of Frank's work is taken from my article in *Ann. Probab.* 21(1993):593-607
2. P. Lévy. *Processus Stochastiques et Mouvement Brownien,* sect. 53-54. Paris: Gauthier-Villars (1948).
3. M. Yor. Etude asymptotique des nombres de tours de plusieurs mouvements browniens complexes corrélés. In *Random Walks, Brownian Motion and Interacting Particle Systems,* eds. R. Durrett and H. Kesten, pp. 441-55. Boston: Birkhäuser (1991).
4. E. S. Andersen. On the number of positive sums of random variables. *Skand. Aktuarietidskr.* 32(1949):27-36. On the fluctuations of sums of random variables I and II. *Math. Scand.* 1(1953):263-85 and *Math. Scand.* 2:(1954):195-223. Correction in *Math. Scand.* 2(1954):193-94.
5. F. Pollaczek. Problèmes stochastiques posés par le phénomène de formation d'une queue d'attente à un guichet et par des phénomènes apparentés. *Mém. Soc. Math. France* 136, eq. 7.16.

6. I have not been able to confirm with certainty that these are the correct years.

7. S. Kakutani. Two-dimensional Brownian motion and harmonic functions. *Proc. Imp. Acad. Japan* 20(1944):706-14.

8. J. L. Doob. Semimartingales and subharmonic functions. *Trans Amer. Math. Soc.* 77(1954):86-121.

9. G. A. Hunt. Markoff processes and potentials I-III. *Ill. J. Math.* 1(1957):44-93 and 316-19, and 2(1958):294-319.

10. T. E. Harris. Diffusion with "collision" between particles. *J. Appl. Probab.* 2(1965):323-38.

11. There are too many related articles by Dobrushin to cite them all. Two examples are (1) R. L. Dobrushin. Markov processes with a large number of locally interacting components: existence of a limit process and its ergodicity. *Problems Inform. Transmission* 7(1971):149-64 and (2) R. L. Dobrushin. Markov processes with many locally interacting components—the reversible case and some generalizations. *Problems Inform. Transmission* 7(1971):235-41.

12. However, the reader is referred to D. Griffeath, "Frank Spitzer's pioneering work on interacting particle systems (*Ann. Probab.* 21(1993):608-21) for a very fine survey of Frank's contributions. Excellent treatments of interacting particle systems in book form can be found in T. M. Liggett, *Interacting Particle Systems* (Berlin: Springer-Verlag, 1985) and R. Durrett, *Lecture Notes on Particle Systems and Percolation* (Pacific Grove: Wadsworth, 1988).

13. For a partial survey see A. De Masi and E. Presutti, *Mathematical Methods for Hydrodynamic Limits*, Lecture Notes in Mathematics, vol. 1501. Berlin: Springer-Verlag (1991).

14. See for instance R. H. Schonmann. Theorems and conjectures on the droplet-driven relaxation of stochastic Ising models. In *Probability and Phase Transition*, ed. G. Grimmett, pp. 265-301. Dordrecht: Kluwer Academic Publishers (1994).

15. See for instance R. Durrett. Ten lectures on particle systems. In *Lecture Notes in Mathematics*, vol. 1608, pp. 97-201. Berlin: Springer-Verlag (1995).

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