James B. Serrin

1926–2012

BIOGRAPHICAL *Memoirs*

A Biographical Memoir by
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JAMES BURTON **SERRIN**

November 1, 1926–August 23, 2012

Elected to the NAS, 1980

By Donald G. Aronson[1] and Hans F. Weinberger[1]

James Burton Serrin, known to many friends and colleagues as Jim and to many others as James, was a preeminent American mathematician who made fundamental contributions to fluid dynamics, minimal surface theory, thermodynamics, and partial differential equations.

Jim was born in Chicago, IL, and brought up in nearby Evanston. His father James Burton Serrin, Sr., was an insurance broker whose business suffered greatly in the crash of 1929 and the subsequent depression. His mother, Helen Elizabeth Wingate, was a homemaker. His brother Richard Wingate Serrin, two years younger than Jim, has become a talented and established artist living and working in Florence, Italy. Their mother was rather strict. For example,[2] she did not approve of eyeglasses, and despite Jim's extreme nearsightedness, she would not permit him to wear them. She finally relented when he was in junior high school, although she still banned them from the house.

Jim graduated from Evanston Township High School in 1944. Pursuing his interest in mathematics and science, he entered Northwestern University in 1944 with a major in electrical engineering. The family moved to Kalamazoo, Michigan shortly thereafter, and he transferred to Western Michigan College (now Western Michigan University) that same year. He graduated with a B.A. degree in 1947.

Jim did his graduate work at Indiana University in Bloomington. Concerning his experience at IU, he wrote: "In the years 1948 to 1950, [I] had the unique opportunity of attending lecture courses on elliptic differential equations given by Professors Eberhard Hopf and David Gilbarg. These exemplary lectures first awakened [my] interest in this theory and in particular in the subject of the maximum principle" (Pucci and Serrin

1. School of Mathematics, University of Minnesota.

2. Personal communication from Elizabeth Conley.

2007). Under Gilbarg's supervision, Jim studied fluid dynamics and partial differential equations. While still a student, he and Gilbarg collaborated on and published a paper titled "Free boundaries and jets in the theory of cavitation"(Gilbarg and Serrin 1950).

Jim received his Ph.D. in 1951. His thesis "The Existence of Flows Solving Four Free-Boundary Problems" was written under Gilbarg's supervision; and the first part of this work was published in 1952, with the title "Existence theorems for some hydrodynamical free boundary problems," in the first issue of *Journal for Rational Mechanics and Analysis* (Serrin 1952).

After receiving his degree, Jim left IU to take up an appointment as a Fine Instructor of Mathematics at Princeton University. In 1952 he moved to the Massachusetts Institute of Technology as a C. L. E. Moore Instructor. While at MIT he began his seminal work on elliptic partial differential equations.

On September 6, 1952, Jim married Barbara West. He and Barbara had three daughters: Martha (Stack) in 1953, Elizabeth (Conley) in 1957, and Janet (Sucha) in 1967.

In 1954 Jim was appointed assistant professor of mathematics at the University of Minnesota, where he remained for the rest of his career. At that time, the University of Minnesota had two separate departments of mathematics, one in the College of Liberal Arts, and one in the Institute of Technology (IT). Jim joined the IT department, which was led by Stephan Warschawski, who was encouraged by his dean to establish a strong group of mathematicians who were interested in partial differential equations and their physical applications. His recruits included Eugene Calabi, Arthur Milgram, Paul Rosenbloom, and Hidehiko Yamabe, who were then at their prime, and a number of very good younger faculty.

Jim and Barbara soon established themselves in Minneapolis as wonderful hosts both to local friends and visitors. They eventually acquired a large and gracious home, which they filled with antiques, art, and books. They entertained frequently, and were noted for their warm hospitality. Jim's artist brother Richard painted in the classical style, and many of his pictures were displayed in Jim and Barbara's home. Barbara was a gracious hostess and a strong support for Jim all his life. She followed him in death a year after Jim had passed away.

Jim quickly displayed a real knack for bringing out the best in talented young mathematicians. During his early career at Minnesota, he published strong papers coauthored with Don Aronson, with Howard Jenkins, and with Norman Meyers. In 1980, Lamberto

> James had a strange relationship with airplanes. He knew enough about fluid dynamics to understand that both the theory, which was used to describe airplanes' flight, and the numerical techniques needed to make computations for building the airplanes, were rather shaky.

Cesari introduced James to his student Patrizia Pucci, and they began a fruitful collaboration that lasted until Jim's death. While he also collaborated with former students such as Giovanni Leoni, Kevin McLeod, and Henghui Zou, Jim worked with many other young mathematicians as well. Near the end of his life, he collaborated with Alberto Farina to produce three papers, one of which appeared after his death. Jim was also collaborating with Chi-Sing Man on a book on thermodynamics when he died. This book is being finished by his coauthor.

James had a strange relationship with airplanes. He knew enough about fluid dynamics to understand that both the theory which was used to describe airplanes' flight, and the numerical techniques needed to make computations for designing the airplanes were rather shaky. As a result Jim refused to fly for many years. He was ultimately forced to change his mind, in large part due to the pressure of overseas invitations and the demise of ocean liners.

In addition, when the Serrins first came to Minnesota they bought a house that happened to be near the then lightly used airport. As air travel grew, and the resulting noise became unbearable, the Serrins bought a beautiful house farther away from the airport. However, as air travel grew some more, a new runway appeared in a direction which put the new house on the main landing path. The noise had followed Jim. He fought back valiantly by joining a group that was trying to get the airport moved about 20 miles from town, with a good connection by a dedicated train. This seemed like a reasonable solution, but it would be very costly to Northwest Airlines, which had a large investment in the present location. So Northwest pulled a few strings, and the proposal was defeated. Jim was very disappointed, but this display of corporate power did nothing to shake his belief in conservative politics.

The exceptional quality of James Serrin's research was greatly appreciated in many quarters, and he was much in demand for invited lectures and colloquia. He gave invited addresses at the International Congress of Mathematicians in Nice in 1970 ("Boundary curvatures and the solvability of Dirichlet's problem") and in Warsaw in 1983 ("The structure and laws of thermodynamics"). He was awarded honorary degrees by the University of Sussex (1972), the University of Ferrara (1992), the University of Padua

(1992), and the University Francois-Rabelais at Tours (2005). In 1973, he received the George David Birkhoff Award in Applied Mathematics from the American Mathematical Society, which cited Jim "...for his fundamental contributions to the theory of nonlinear partial differential equations, especially his work on existence and regularity theory for nonlinear elliptic equations, and [for] applications of his work to the theory of minimal surfaces in higher dimensions." The University of Indiana gave him its Distinguished Alumni Award in 1979.

Jim was elected to the National Academy of Sciences in 1980, to the American Academy of Arts and Sciences in 1984, and as a fellow of the American Association for the Advancement of Science in 1980. In 1995 he was elected a foreign member of the Finnish Academy of Sciences. Jim was a member of the Council of the American Mathematical Society from 1972 to 1974 and was chairman of the Society for Natural Philosophy from 1969 to 1970. To commemorate his 60th birthday in 1986, a series of papers dedicated to him in the *Archive for Rational Mechanics and Analysis* were collected in the volume *Analysis and Continuum Mechanics* and published by Springer Verlag in 1987. Conferences to celebrate Jim's 65th birthday were held in Minnesota and Ferrara. Conferences in honor of his 75th and 80th birthdays were held in Perugia. Jim held visiting professorships at Stanford, Chicago, Johns Hopkins, Sussex, the Mittag-Leffler Institute, Naples, Perugia, Modena, Oxford, and the National Science Council of Taiwan.

Jim's dissertation on cavitation flows led him to some fundamental results in the study of the Navier-Stokes equations, about which he wrote a number of papers during his 1951–52 stay at Princeton. One of them, "Comparison theorems for subsonic flows," (Serrin 1954) extended a technique of Gilbarg from incompressible flows to compressible flows. He also collaborated with Gilbarg on another paper, "Uniqueness of axially symmetric subsonic flow past a finite body" (Gilbarg and Serrin 1955) which represented the first such result for a three-dimensional flow.

When Jim came to Minnesota in 1954, he was interested in extending the known results of S. N. Bernstein and of W. Feller to as large a class of linear nondivergence-form elliptic equations as possible. Jim showed that in two dimensions, Harnack's inequality can be obtained when one assumes only that the coefficients are continuous. For higher dimensions, one had to assume either the Dini continuity of the highest-order coefficients or an a priori bound for the gradient of the solution (Serrin 1956). The extension to higher dimensions of the result without an extra condition did not appear until the work of M. V. Safonov was published 24 years later (Safonov 1980).

In 1956 Jim published a joint paper with Gilbarg. Here the Harnack inequality was used to study the local behavior of solutions of linear elliptic partial differential equations in arbitrarily many dimensions. The equations may or may not be in divergence form, and the continuity assumptions on the coefficients are quite weak and are consistent with the known existence and uniqueness theory. The results include an extended maximum principle for solutions with a singular point, asymptotic and limit behavior of solutions, a Liouville theorem, and the characterization of the fundamental solution. The effect of the continuity properties of the coefficients on the behavior of solutions were illustrated in the paper by several illuminating examples (Gilbarg and Serrin 1956).

Olga Ladyzhenskaya and David Gilbarg
(Photo courtesy of the Serrin family.)

Another outcome of this effort was a joint work with Jim's younger colleague Norman Meyers, which produced fundamental new results about the exterior Dirichlet problem for uniformly elliptic second-order linear homogeneous and inhomogeneous equations (Meyers and Serrin 1960). In particular, it was shown that if the equation is uniformly elliptic and has smooth coefficients, and if the boundary is smooth, then either the Dirichlet problem with the condition that the solution be bounded, or the Dirichlet problem with the condition that the solution have a specified limit at infinity, is well-posed. This was Jim's first of many collaborations with younger mathematicians.

In 1957, J. F. Nash and E. de Giorgi independently proved that all weak solutions of divergence-form elliptic equations with bounded and measurable coefficients were Hölder continuous. This finding shifted the focus of many mathematicians to divergence-form equations, and it inspired Jürgen Moser to produce a greatly simplified proof in 1960. In 1961, Moser found that his method of proof also showed that solutions of divergence-form elliptic equations with bounded and measurable coefficients satisfy Harnack's inequality, which in turn implies the Hölder continuity. The Moser works were studied by the partial-differential-equations group at Minnesota in an extensive series of seminars.

Jim showed in a 1964 paper that Moser's results could be extended to the weak solutions of an elliptic quasilinear equation of the form

$$divA\left(x,u,Du\right)=B\left(x,t,u\right) \qquad (1)$$

provided that for some constants $\alpha>1$ and $a>0$ and measurable functions *b*(*x*), *c*(*x*), *d*(*x*), *e*(*x*), *f*(*x*) and *g*(*x*), the vector-valued function ***A*** and the scalar function ***B*** satisfy the growth conditions

$$|A\left(x,u,\mathrm{p}\right)|\leq a\,|\,\mathrm{p}\,|^{\alpha\text{-}1}+b\left(x\right)|\,u\,|^{\alpha-1}+e\left(x\right)$$

$$|B\left(x,u,\mathrm{p}\right)|\leq c\left(x\right)|\,\mathrm{p}\,|^{\alpha-1}+d\left(x\right)|\,u\,|^{\alpha-1}+f\left(x\right) \qquad (1')$$

$$\mathrm{p}\cdot A\left(x,u,\mathrm{p}\right)\geq a^{-1}\,|\,\mathrm{p}\,|^{\alpha}-d\left(x\right)|\,u\,|^{\alpha}-g\left(x\right).$$

A remarkable fact is that local boundedness, the Harnack inequality, and Hölder continuity were obtained without any explicit assumption that the equation is elliptic. While Jim's finding overlapped with some of the results obtained by O. A. Ladyzhenskaya and N. N. Uraltseva a few years earlier, it was an important step forward (Serrin 1964).

The extension from results for linear equations to quasilinear equations of the form (1) with the growth conditions (1') is an art that James practiced during the rest of his career. He soon showed that the results on isolated singularities and the Liouville theorem for linear equations can be extended to this class (Serrin 1965a). Jim also showed that if *u* is a solution of (1) on a set $D-Q$, where *D* is open and *Q* is a compact set whose $(n-1)$-dimensional Hausdorff measure is zero, then *u* can be defined on *Q* so that it is a solution in all of *D*. (Serrin 1965b).

The equation for a nonparametric minimal surface has the form (1), and beginning in 1963, a series of joint papers with Jim's young colleague Howard Jenkins produced remarkable results for this and similar problems (Jenkins and Serrin 1968a, 1968b, 1966, 1963). In particular, they showed that the Dirichlet problem for the minimal surface equation in more than two dimensions is well-posed if and only if the boundary has nonnegative mean curvature.

A paper with Don Aronson (Aronson and Serrin 1967) extended Moser's technique to prove the Harnack principle and derive various other results for a large class of quasi-linear parabolic equations.

A similar but different extension of the theory of linear equations pertains to the class of uniformly elliptic quasilinear equations of the form

$$A(x,u,Du)D^2u = B(x,u,Du), \tag{2}$$

which does not have the divergence structure of (1). The minimal surface equation can be written in this form, as well as in the form (1). In a pair of papers, Serrin developed an extension to arbitrarily many dimensions of the results of S. N. Bernstein and J. Leray regarding the solutions and local behavior of the Dirichlet problem in two dimensions (Serrin 1969, 1967). He also sharpened their results by removing various of their regularity assumptions on the coefficients of the equations and replacing them with quite general structure assumptions. Bernstein had already observed that the Dirichlet problem was not generally solvable for certain equations, such as the mean curvature equation, even when the domain was convex and sufficiently small. In his two papers Jim gave a precise characterization of those domains for which the Dirichlet problem is solvable. A great deal of additional research has grown out of this work.

Beginning in the 1950s and thereafter, many mathematicians were exploring the ramifications of the theory of distributions introduced by Sergei Sobolev and elaborated on by Laurent Schwartz. In particular, some strong solutions and some weak solutions for elliptic problems had been constructed, but the relation between the two sets was unclear unless the boundary was smooth. In 1964, Jim and his young colleague Norman Meyers published a two-page paper, titled "*H* = *W*" which showed that in fact there is no difference between weak and strong solutions (Meyers and Serrin 1964).

Clifford Truesdell, one of Jim's teachers at Indiana, had agreed to edit one volume of the new *Handbuch der Physik*, and he asked Serrin to write the article on fluid mechanics for this volume. This was a major task that took much of Jim's time and energy during his early days at Minnesota. The labor ended in 1959 with the publication of the *Handbuch* article (Serrin 1959), which is still the definitive text on the subject, and it was followed by a number of papers that were byproducts of this work.

In 1961, there were two different constructions of weak solutions of the initial value problem for the Navier-Stokes equations, one by E. Hopf (1951), and one by A. A. Kiselev and O. A. Ladyzhenskaya (1957). In research he conducted at that time (Serrin 1962), Jim showed that the Hopf solution is infinitely differentiable in space and Lipschitz continuous in time. He did this by using the Bochner space with norm

$$\left\|g(x,t)\right\|_{p,p'} := \left[\int \left\{\int |g(x,t)|^p \, dx\right\}^{p'/p} dt\right]^{1/p'}.$$

While Serrin's condition

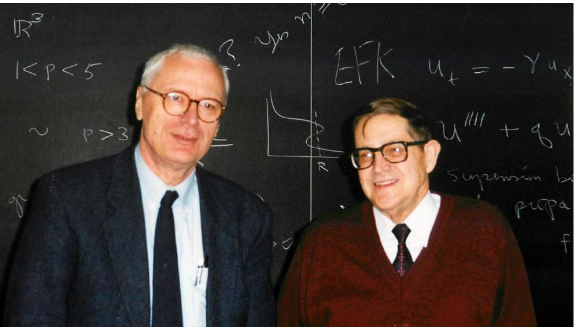
$$\frac{1}{p}+\frac{1}{p'}<1$$

has been strengthened to permit equality, it remains the standard norm in the treatment of the Navier-Stokes equations. J. L. Lions and G. Prodi had proved a uniqueness theorem for weak solutions of the two-dimensional Navier-Stokes equations, but because the Hopf solution did not satisfy one of its conditions, one could not conclude that the Kiselev-Ladyzhenskaya solution is smooth. Jim remedied this situation in by strengthening the Lions-Prodi theorem to one which can be used to show that the Kiselev-Ladyzhenskaya solution is equal to the smooth Hopf solution (Serrin 1963). Because in two dimensions the Kiselev-Ladyzhenskaya solution is defined for all time (Ladyzhenskaya 1959), Jim's work finished the existence theory for the initial value problem in two dimensions. The corresponding problem in three dimensions is still open as of this writing (January 2016).

In 1971, Serrin published a paper in which he showed that if Ω is a smooth domain, then a necessary and sufficient condition for the existence of a solution of the overdetermined problem

$$\begin{aligned} \Delta u &= -1 \ \ in\ \ \Omega \\ u &= 0 \ \ on\ \ \partial\Omega \\ \partial u/\partial n &= \text{constant on } \partial\Omega \end{aligned} \tag{3}$$

is that Ω is a ball. The important part of this paper was the method employed. This method involved a reflection of the domain about a family of planes moving perpendicularly to themselves, and the method also featured an improved version of the strong



Bert Pelletier and James Serrin
(Photo courtesy of the Serrin family.)

maximum principle. It turned out that the first part of this procedure had been used by A. D. Aleksandrov in greatly generalizing the known fact that a surface with constant positive Gaussian curvature or mean curvature is a sphere (Aleksandrov 1962). The second part involved a strengthened form of the Hopf boundary point lemma, and that part was new. While the Serrin paper emphasized the simple problem (3), it also showed how the results could be extended to a class of quasilinear equations.

A famous 1979 paper of B. Gidas, W.-M. Ni, and L. Nirenberg (Gidas, Ni, Nirenberg 1979) simplified and generalized the proof of the moving planes method. More importantly, they applied this method to the domains bounded by level surfaces of a positive solution of the semilinear equation of the form

$$\Delta u + f(u) = 0,$$

which vanishes at infinity to show that u must be radially symmetric (Gidas, Ni, and Nirenberg 1979). There was much interest at the time in such solutions, called "ground states," which arose from the Yang-Mills theory of quantum mechanics. The Gidas-Ni-Nirenberg paper reduced the study of these solutions to the study of ordinary differential equations. The radial symmetry results of that paper were not confined to semilinear equations but also were applicable to a class of nonlinear elliptic equations.

Because Jim and L. A. Pelletier had been studying positive solutions of some semilinear elliptic equations for some time, the Gidas-Ni-Nirenberg paper aroused Jim's interest in radially symmetric ground states. In 1985 and 1986, he wrote a joint paper with Wei-Ming Ni, who was by then a young colleague at Minnesota, which presented conditions on the vector-valued function A and the scalar-valued function f under which the quasilinear elliptic equation

$$div\left[A(|Du|)Du \right] + f(u) = 0 \tag{4}$$

has or does not have an axially symmetric ground state (Ni and Serrin 1985). He also collaborated with other young mathematicians, including B. Franchi and E. Lanconelli, on such problems.

James with Patrizia Pucci
(Photo courtesy of the Serrin family.)

Because equation (4) does not necessarily satisfy the conditions of the Gidas-Ni-Nirenberg paper, the nonexistence of a radially symmetric ground state does not imply the nonexistence of a ground state, and uniqueness among radially symmetric ground states does not imply the uniqueness of a ground state. Jim worked diligently to overcome this gap. Success came in a 1999 paper, coauthored with his former student Henghui Zou, which showed that under a few conditions on f the basic quasilinear elliptic equation (4) has a unique ground state if a certain integral involving A and f is infinite; and that this ground state is radially symmetric. Moreover, if the integral condition is violated, then every ground state vanishes outside a bounded set. In the latter case, either the only ground state is zero; or there are infinitely many ground states, each of which has support on an arbitrary union of finitely many disjoint balls, and the solution is radially symmetric in each ball (Serrin and Zou 1999).

We have already mentioned Jim's extensive collaborations with a large number of younger mathematicians. Some of these partnerships endured for many years. For example, his collaboration with Patrizia Pucci, which lasted for over 40 years, began with work on the topological properties of the sets of solutions that could be obtained from the mountain pass theorem of A. Ambrosetti and P. H. Rabinowitz (Ambrosetti and Rabinowitz 1973). It soon branched out into other areas of mathematics, including the study of ground states.

Clifford Truesdell between Barbara and James Serrin
(Photo by Patricia Puzzi, courtesy of the Serrin family.)

An important tool in the study of solutions of semilinear equations in star-shaped domains is a well-known generalization by S. I. Pohozaev of Rellich's identity (Pohozhaev 1965). An extension of this identity to radially symmetric solutions of quasilinear equations was used in Jim's joint paper with Ni (Ni and Serrin 1985). In 1986, Jim and Pucci found a full extension to a family of systems of quasilinear elliptic equations whose leading terms are given by an Euler equation (Pucci and Serrin 1986). A particular application of this idea permits the treatment of equations that involve the polyharmonic operator Δ^k.

The proof of the results in Serrin and Zou's paper (1999) heavily relied on a very strong maximum principle for the quasilinear equation (4), which appeared in a paper written jointly by Jim, Pucci, and Zou (Pucci, Serrin, and Zou 1999). Many of the results obtained by Jim and Pucci entailed the development and use of new and ever more delicate forms of the maximum principle for quasilinear differential equations and inequalities. James and Pucci combined many of the results of this joint effort into the book *The Maximum Principle* (Pucci and Serrin 2007), which is now the definitive reference on the subject.

Another topic to which James devoted a great deal of time and effort was rational thermodynamics. When he arrived at Indiana University in 1948, the Department of Mathematics was already a center of "rational mechanics," a term that was introduced by Newton. The core of the group consisted of David Gilbarg, Eberhard Hopf, and T. Y. Thomas, all of whom were accomplished pure mathematicians who shared a strong interest in obtaining significant results about the partial differential equations that arise from models of continuum mechanics. Jim chose to work with Gilbarg on the Navier-Stokes system for classical fluid dynamics. In 1950, Thomas, the head of the department, hired Clifford Truesdell, a polymath with deep interest in and knowledge of the history and philosophy of mathematics. Truesdell had a very magnetic personality and a rather large circle of colleagues and disciples who were sympathetic to his views. Jim was drawn into this circle, and it had a lasting influence on his career.

Roger Fosdick
(Photo courtesy of the Serrin family.)

To encourage work in continuum mechanics and in the analysis of the resulting nonlinear partial differential equations, Truesdell and Thomas founded the *Journal of Rational Mechanics and Analysis*, and Jim's 1952 dissertation was the first paper to appear in this publication.

In 1957, a new department head at Indiana removed Truesdell from the editorship of the *Journal of Rational Mechanics and Analysis*. and changed the name of the journal to *Journal of Mathematics and Mechanics.* Truesdell persuaded Springer Verlag to continue the journal, but for legal reasons its name had to be changed to *Archive for Rational Mechanics and Analysis*. Jim served on the editorial board of this new journal from that time on, and he was its coeditor from 1967 to 1985. Truesdell also founded the Society for Natural Philosophy in order to encourage further work in rational mechanics and in related areas of the mathematical and physical sciences. Jim was the fourth chairman of this society.

Because the viscosity in classical fluid dynamics and the internal friction in the mechanics of deformable bodies involve the conversion of mechanical work into heat, Truesdell realized that rational mechanics would have to include thermodynamics. However, unlike most of classical thermodynamics, rational mechanics would also have to include processes that are far from being reversible. While Serrin's *Handbuch der Physik* article (Serrin 1959) certainly needed to discuss some of these topics, he seriously entered the fray some years later, in 1975, through a joint paper with Roger Fosdick, a colleague in the University of Minnesota's Department of Aerospace Engineering and Mechanics. Among other contributions, this paper noted the "curious generalization" that all the thermodynamic results that one could obtain by assuming that the total energy of a body is neither increased nor decreased could already be deduced from the weaker hypothesis that the total energy is not increased (Fosdick and Serrin 1975).

The only irreversible processes which can be handled by classical thermodynamics are uniformly small perturbations of equilibria. Because most interesting problems in thermodynamics treat cycles which are neither reversible nor close to an equilibrium, a new idea was clearly needed.

In 1978 and 1979 Jim wrote two fundamental papers (Serrin 1978) and Serrin (1979) which addressed this issue. The basic idea was to turn a remark of Gibbs into a new function $Q(P;L)$, which James called the accumulation function of a cyclic process *P*. Here *L* is a local property called hotness, which is often given by an absolute temperature. $Q(P;L)$ is defined as the integral of all heat inputs at hotness levels $L' \leq L$ during the cycle *P*. For large *L*, $Q(P;L)$ is just the net input of heat during the cycle *P*. The classical first law of thermodynamics implies that this quantity is positive when the work *W*(*p*) done by *P* is positive. Serrin showed (see, e.g., Combined Laws on p.15 of (Serrin, 1986)) that the pair of statements

$$Q(P;L) > 0 \text{ for sufficiently large } L \text{ when } W(P) > 0$$

$$Q(P;L) < 0 \text{ for some } L \text{ unless } Q(P;L) = 0 \text{ for all } L.$$

is equivalent to the first and second laws of thermodynamics in the classical case, but also extends these laws to cycles which need not be reversible or close to an equilibrium. Serrin's Accumulation Theorem shows that if the hotness can be measured by the absolute temperature of a perfect gas thermometer, then the negativity of the accumulation function $Q(P;L)$ can be replaced by the negativity of a single integral.

Further extensions of such results were given in (Coleman, Owen, and Serrin 1981), and in (Man and Massoud, 2010). Before he knew of (Serrin 1978) or (Serrin 1979), M. Silhavy independently defined what he called the *heat measure*, which can be viewed as a Borel measure version of the accumulation function, and derived results which are similar to those of Serrin. See, e. g., (Šilhavý 1986). Given these and other contributions, there is a large field, called “neoclassical thermodynamics” by the reviewer of the monograph (Šilhavý 1986) in which irreversible thermodynamics problems far from equilibrium can be handled.

Because Serrin's formulation gives such a vast extension of the problems to which thermodynamics can be applied, Truesdell dedicated the second edition of his book *Rational Thermodynamics* to “James Serrin who with new, sharp instruments has cleared and firmed old grounds, opened new prospects, built new structures” (Truesdell 1991).

We have provided a brief survey of some of Jim’s most significant research accomplishments, but it does not really convey the full depth and breadth of his contributions; a more comprehensive picture can be gleaned from *James Serrin, Selected Papers* (ed. Pucci, Radilescu, and Weinberger 2014). Jim was a truly great mathematician whose influence continues to be felt throughout the scientific world. His passing represents a great loss to his friends and colleagues, and to the larger mathematical community.

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