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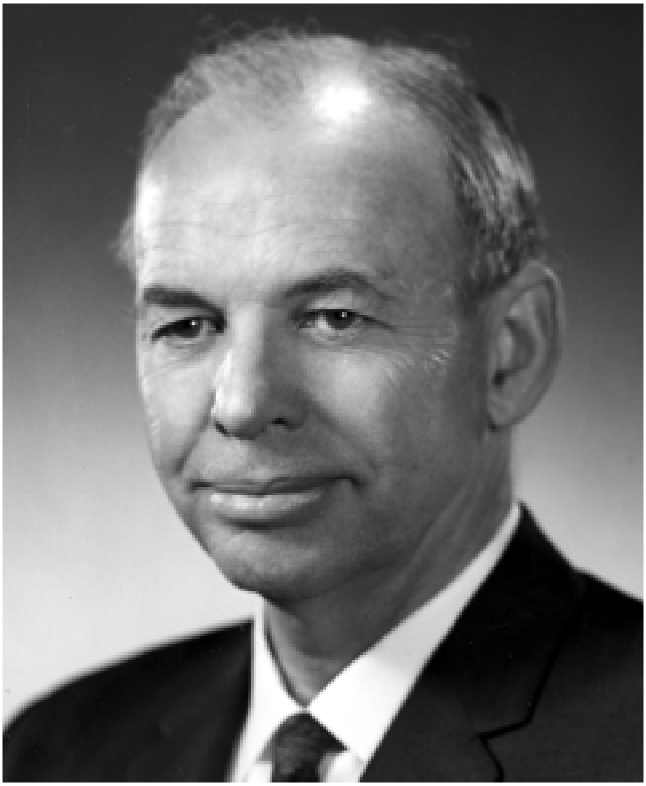

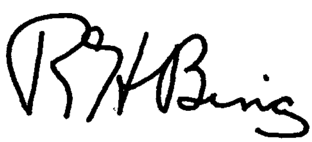

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R. H. BING

October 20, 1914–April 28, 1986

BY MICHAEL STARBIRD

R. H. BING LOVED TO work on problems in topology, perhaps because he was consummately successful at solving them. He was a mathematician of international renown, having written seminal research papers in general and geometric topology. The Bing-Nagata-Smirnov metrization theorem is a fundamental result in general topology that provides a characterization of which topological spaces are generated by a metric. His work and the methods he used in the study of the geometric topology of 3-dimensional space were so seminal and distinctive that that area of investigation is often referred to as Bing-type topology. His leadership both in research and in teaching resulted in his serving as president both of the American Mathematical Society and the Mathematical Association of America. However, we who knew him personally remember him most for his zest for life that infected everyone around him with a contagious enthusiasm and good humor. So this paper celebrates a life well lived, a life whose joy came partly from significant contributions to topology and partly from an overflowing *joie de vivre.*

A VIGNETTE ON BING'S DEVOTION TO MATHEMATICS

It was a dark and stormy night, so R. H. Bing volunteered to drive some stranded mathematicians from the fogged-in Madison airport to Chicago. Freezing rain pelted the windscreen and iced the roadway as Bing drove on—concentrating deeply on the mathematical theorem he was explaining. Soon the windshield was fogged from the energetic explanation. The passengers too had beaded brows, but their sweat arose from fear. As the mathematical description got brighter, the visibility got dimmer. Finally, the conferees felt a trace of hope for their survival when Bing reached forward—apparently to wipe off the moisture from the windshield. Their hope turned to horror when, instead, Bing drew a figure with his finger on the foggy pane and continued his proof—embellishing the illustration with arrows and helpful labels as needed for the demonstration.

Two of Bing's mathematical colleagues Steve Armentrout and C. E. Burgess independently recalled versions of this memorable evening. Those of us who knew Bing well avoided raising mathematical questions when he was driving.

R. H. Bing started and ended in Texas. He was born on October 20, 1914, in Oakwood, Texas, and there he learned the best of the distinctively Texas outlook and values. What he learned in Oakwood guided him clearly throughout his life. He had a strong Texas drawl, which became more pronounced proportionate to his distance from Texas; and he spoke a little louder than was absolutely necessary for hearing alone. He might be called boisterous with the youthful vigor and playful curiosity that he exuded throughout his life. He was outgoing and friendly and continually found ways to make what he did fun. You could hear him from down the hall laughing with his TA's while grading calculus exams or doing other work that deadens most people. He did not sleep well and when he woke at 4:00 a.m., he would get up and work. He especially enjoyed working on things requiring loud hammering at that hour on the grounds that if you are going to be up at 4, the family should know about it. He practiced the traditional Texas value of exercising independent judgment, both in general matters and in matters mathematical. He treated people kindly and gently—unless he knew them, in which case it was more apt to be kindly and boisterously.

Both of Bing's parents were involved in education. His mother was a primary teacher and his father was the superintendent of the Oakwood School District. Bing's father died when R. H. was five, so Bing most remembered his mother's impact on his character and interests. Bing attributed his love for mathematics to his mother's influence. He recalled that she taught him to do mental arithmetic quickly and accurately and to enjoy competition both physical and mental.

After high school Bing enrolled in Southwest Texas State Teachers College in San Marcos (now Southwest Texas State University) and received his B.A. degree in 1935 after two and a half years there. Later in life Bing was named as the second distinguished alumnus of Southwest Texas State University. The first person so honored was Lyndon Baines Johnson. Bing's college education had prepared him as a high-school mathematics teacher. He also was a high-jumper on the track team and could jump his own height—which was over 6 feet.

Bing's final academic position was as the Mildred Caldwell and Blaine Perkins Kerr Centennial Professor in Mathematics at the University of Texas at Austin, but his first academic appointment was as teacher at Palestine High School in Palestine, Texas. There his duties included coaching the football and track teams, teaching mathematics classes, and teaching a variety of other classes, one of which was typing. His method of touch-typing involved anchoring his position over the keys by keeping some constant pressure on his little fingers. This habit was hard to break, apparently, because later he said that when he used an electric typewriter or computer keyboard (neither of which he did often) he tended to produce large numbers of extraneous "a's."

Nowadays one frequently hears complaints about a school system that gives the football coach the added assignment of teaching a mathematics class. One wonders if those football boosters of a bygone day in Palestine complained to the local school board about a real mathematics teacher coaching the football team.

In an effort to improve public school education in the 1930s, the Texas legislature approved a policy whereby a teacher with a master's degree would receive more pay than a teacher with a bachelor's degree. So, many teachers saved and scrimped during the nine-month session and went to summer school during the three summer months in an effort to upgrade their talents and their salaries. Bing was among them.

R. H. began public school teaching in 1935 and began taking summer school courses at the University of Texas at Austin. There he met Mary Blanche Hobbs, whom he married in 1938. R. H. and Mary enjoyed a long and happy marriage. They had four children: Robert Hobbs Bing, 1939; Susan Elizabeth Bing, 1948; Virginia Gay Bing, 1949; and Mary Pat Bing, 1952. His wife, Mary, all their children, their children's spouses, and their grandchildren still have fond memories of Bing.

The same year as his marriage he earned a master of education degree from UT. During one summer there Bing took a course under the late Professor R. L. Moore, also a member of the National Academy of Sciences. Moore was inclined to deprecate the efforts of an older student such as Bing was, so Bing had to prove himself. But he was equal to the task.

Bing continued to take some summer courses while teaching in the high schools. In 1942 Moore was able to get Bing a teaching position at the university, which allowed him to continue graduate study to work toward a doctorate and to try his hand at research.

An unofficial rating scheme sometimes used by R. L. Moore and his colleagues went something like this: You could expect a student with Brown's talents and abilities every year; you could expect a student with Lewis's talents and abilities once every 4 years; but a student with Smith's talents and abilities came along only once in 12 years. Bing's talents and abilities threw him in the 12-year class, or in an even higher class, since he was one of the most distinguished mathematicians ever to have received his degree from the University of Texas at Austin. Several of Moore's later graduate students have written that in the days after Bing, Moore used to judge his students by comparing them with Bing—not to their advantage.

Bing received his Ph.D. in 1945, writing his dissertation on planar webs. Planar webs are topological objects now relegated to the arcana of historical topological obscurity. The results from his dissertation appeared in one of his earliest papers (1946) in the *Transactions of the American Mathematical Society*. He told us that the *Transactions* had sent him 50 reprints at the time and if we were interested we could have some because he still had 49 or so left.

But Bing did not have long to wait for recognition of his mathematical talent. He received his Ph.D. degree in May 1945, and in June 1945 he proved a famous, longstanding unsolved problem of the day known as the Kline sphere characterization problem (1946). This conjecture states that a metric continuum in which every simple closed curve separates but for which no pair of points separates the space is homeomorphic to the 2-sphere.

When word spread that an unknown young mathematician had settled this old conjecture, some people were skeptical. Moore had not checked Bing's proof since it was his policy to cease to review the work of his students after they finished their degrees. Moore believed that such review might tend to show a lack of confidence in their ability to check the work themselves. So when a famous professor wired Moore asking whether any first-class mathematician had checked the proof, Moore replied, "Yes, Bing had."

Primarily because of the renown among mathematicians generated by his having solved a famous conjecture, Bing was offered positions at Princeton University and at the University of Wisconsin, Madison. Moore naturally wrote letters of recommendation. One comment he made was that, although the Kline sphere characterization problem was a much better known topic than that of planar webs, Moore felt that it was Bing's work on planar webs that demonstrated that Bing had the mathematical strength to be an outstanding mathematician.

One of the leading topologists of the time was at Princeton, but Bing did not wish to follow in anyone's footsteps, so in 1947 he accepted a position at Wisconsin. He remained at Wisconsin for 26 years except for leaves: one at the University of Virginia (1949-50), three at the Institute for Advanced Study in Princeton (1957-58, 1962-63, 1967), one at the University of Texas at Austin (1971-72), and brief teaching appointments elsewhere. He returned to the University of Texas at Austin in 1973; but it was during his tenure at the University of Wisconsin, Madison, that his most important mathematical work was done and his prominent position in the mathematical community established.

Bing's early mathematical work primarily concerned topics in general topology and continua theory. He proved theorems about continua that are surprising and still central to the field. Among these results is Bing's characterization of the pseudo arc as a homogeneous indecomposable, chainable continuum (1948). The result that the pseudo arc is homogeneous contradicted most people's intuition about the pseudo arc and directly contradicted a published but erroneous "proof" to the contrary. Bing continued to do some work in continua theory throughout his career—including directing a Ph.D. dissertation in the subject at UT in 1977.

Around 1950 one of the great unsolved problems in general topology was the problem of giving a topological characterization of the metrizability of spaces. In 1951 Bing gave such a characterization in his paper "Metrization of Topological Spaces" in the *Canadian Journal of Mathematics* (1951). Nagata and Smirnov proved similar, independent results at about the same time, so now the result is referred to as the Bing-Nagata-Smirnov metrization theorem. That 1951 paper of Bing's has probably been referred to in more papers than any other of his papers, even though he later was identified with an altogether different branch of topology. Bing's paper enfolds in a manner consistent with an important strategy he practiced in doing mathematics. He always explored the limits of any theorem he proposed to prove or understand. Consequently, he would habitually construct counterexamples to demonstrate the necessity of each hypothesis of a theorem. In this paper Bing proved theorems numbered 1 to 14 interspersed with examples labeled A through H. The impact of this paper came both from his theorems and from his counterexamples. Bing's metrization theorems describe spaces with bases formed from countable collections of coverings or spaces where open covers have refinements consisting of countable collections of sets. He defined and discussed screenable spaces, strongly screenable spaces, and perfectly screenable spaces—terms that have been largely replaced by new terminology. He proved in this paper that regular spaces are metrizable if and only if they are perfectly screenable. (The term perfectly screenable means that the space has an ω_0-discrete basis.)

His metrization theorems hinged strongly on his understanding of a strong form of normality, and certainly one of the legacies of this paper is his definition of and initial exploration of collectionwise normality. This paper contains the theorem that a Moore space is metrizable if and only if it is collectionwise normal. After identifying this important property of collectionwise normality he explored its limits by constructing an example of a normal space that is not collectionwise normal. Bing was known for his imaginative naming of spaces and concepts, but this example enjoys its enduring fame under the mundane moniker of "Example G." Immediately following his description of Example G, Bing included the following paragraph that formed the basis of countless hours of future mathematicians' labors:

> One might wonder if Example G could be modified so as to obtain a normal developable space which is not metrizable. A developable space could be obtained by introducing more neighborhoods into the space [Example G]. However a difficulty might arise in introducing enough neighborhoods to make the resulting space developable but not enough to make it collectionwise normal.

Nowadays if you refer to Bing-type topology, you are referring to a certain style of geometric analysis of Euclidean 3-space that came to be associated with Bing because of the fundamental work he did in the area and the distinctive style with which he approached it. The first paper Bing wrote in this area was titled "A Homeomorphism Between the 3-Sphere and the Sum of Two Solid Horned Spheres" and appeared in the *Annals of Mathematics* in 1952. It contains one of Bing's best-known results, namely that there are wild involutions of the 3-sphere, that is, it is possible to reflect 3-space through a mirror that is not topologically embedded in the same manner as a flat plane. The result in this paper hinges on a method of shrinking geometric objects in unexpected ways. When Bing first worked on the question considered in this 1952 paper, he naturally did not know whether it was true or false. He claimed that he worked two hours trying to prove it was true, then two hours trying to prove it was false. When he originally worked on this problem, he used collections of rubber bands tangled together in a certain fashion to help him visualize the problem. The mathematics that Bing did is abstract, but he claimed to get ideas about these abstruse problems from everyday objects.

A final note about this problem involves a paper that Bing wrote in 1984 containing one of his last results. If one shrinks the rubber bands in the manner described in Bing's 1952 paper, each rubber band becomes small in diameter but very long. It became interesting to know whether one could do a similar shrinking without lengthening the bands—in other words, could you do the same thing with string as Bing had proved could be done with rubber. Bing's original procedure had been studied by numerous graduate students and research mathematicians for more than 30 years and yet no one had been able to significantly improve Bing's shrinking method. It was left for Bing himself to prove that "Shrinking Without Lengthening" (the title of this final paper) (1988) is possible.

Bing's results in topology grew in number and quality. He proved several landmark theorems and then raised lots of related questions. Because of his habit of raising questions many other mathematicians and students were able to prove good theorems in the areas of mathematics that he pioneered. He emphasized the importance of raising questions in one's papers and encouraged his students and colleagues to do so. He felt that mathematicians who read a paper are often more interested in what remains unknown than they are interested in what has been proved.

The period from 1950 until the mid-1960s was Bing's most productive period of research. He published about 115 papers in his lifetime, most during this period at the University of Wisconsin, Madison. In 1957 alone three of his papers appeared in the *Annals of Mathematics*. These papers concerned decompositions of Euclidean 3-space and the theorem that surfaces embedded in Euclidean 3-space can be approximated by polyhedral surfaces. Later, that result was extended to show that the polyhedral approximation can be constructed to lie "mostly" on one side of the surface being approximated (1963). In a 1958 *Annals* paper he proved that a compact 3-manifold is homeomorphic to S^3 if and only if every simple closed curve is contained in a ball. This theorem was a partial result in an attempt to settle the still unresolved Poincaré conjecture in dimension 3. In the next year the *Annals* published his independent proof of the theorem that 3-manifolds can be triangulated (1959), a result that had recently been proved in a more complicated way by Edwin Moise.

These theorems and many others he proved about tame and wild surfaces in Euclidean 3-space developed the foundations of the investigation of the geometric topology of 3-space. Bing stated and proved basic facts about 3-space and how surfaces can lie in it. He proved that a surface is tame if it can be approximated entirely from the side (1959). He proved that every surface in 3-space contains tame arcs (1962) and every surface in 3-space can be pierced by a tame arc (1962).

Along the way Bing produced many intriguing examples, many with memorable nicknames: "The Bing Sling"—a simple closed curve that pierces no disk (1956); "Bing's Sticky Foot Topology"—a connected countable Hausdorff space (1953); "Bing's Hooked Rug"—a wild 2-sphere in 3-space that contains no wild arc (1961). These examples helped show the limits of what is true.

His research success brought him honors, awards, and responsibilities. He was quickly promoted through the ranks at the University of Wisconsin, becoming a Rudolph E. Langer Research Professor there in 1964. He was a visiting lecturer of the Mathematical Association of America (1952-53, 1961-62) and the Hedrick lecturer for the Mathematical Association of America (1961). He was chairman of the Wisconsin Mathematics Department from 1958 to 1960, but administrative work was not his favorite. He was president of the Mathematical Association of America (1963-64).

In 1965 he was elected to membership in the National Academy of Sciences. He was chairman of the Conference Board of Mathematical Sciences (1966-67) and a U.S. delegate to the International Mathematical Union (1966, 1978). He was on the President's Committee on the National Medal of Science (1966-67, 1974-76), chairman of the Division of Mathematics of the National Research Council (1967-69), member of the National Science Board (1968-75), chairman of the Mathematics Section of the National Academy of Sciences (1970-73), on the Council of the National Academy of Sciences (1977-80), and on the Governing Board of the National Research Council (1977-80). He was a colloquium lecturer of the American Mathematical Society in 1970. In 1974 he received the Distinguished Service to Mathematics Award from the Mathematical Association of America. He was president of the American Mathematical Society in 1977-78. He retired from the University of Texas at Austin in 1985 as the Mildred Caldwell and Blaine Perkins Kerr Centennial Professor in Mathematics. He received many other honors and served in many other responsible positions throughout his career. He lectured in more than 200 colleges and universities in 49 states and in 17 foreign countries.

Bing believed that mathematics should be fun. He was opposed to the idea of forcing students to endure mathematical lectures that they did not understand or enjoy. He liked to work mathematics out for himself and thought that students should be given the opportunity to work problems and prove theorems for themselves. During his years in Wisconsin Bing directed a very effective training program for future topologists. The first-year graduate topology class, which he often taught there, would sometimes number 40 or more students. He directed the Ph.D. dissertations of 35 students and influenced many others during participation in seminars and research discussions.

Bing enjoyed teaching and felt that experiments in teaching were usually successful—not because the new method was necessarily better but because doing an experiment showed an interest in the students, which they appreciated and responded to. Here are a couple of the experiments he tried while teaching at UT. Bing thought that a person who could solve a problem quickly deserved more credit than a person who solved it slowly. He would say that an employer would rather have an employee who could solve two problems in as much time as it took for someone else to solve one. So in some of his undergraduate classes he introduced speed points. For a 50-minute test he gave an extra point for each minute the test was submitted early. He noticed that often the people who did the work the quickest also were the most accurate. Speed points were somewhat popular and sometimes he would let the class vote on whether speed points would be used on a test. Another experiment in test giving was not popular. One day Bing prepared a calculus test that he realized was too long. Instead of deleting some questions, however, he decided to go ahead and give the test, but as he phrased it, "Let everyone dance to the tune of their own drummer." That is, each person could do as many or as few of the problems as he or she wished and would be graded on the accuracy of the problems submitted. The class was quite angry when the highest score was obtained by a person who had attempted only one problem.

In the 1971-72 school year Bing accepted an offer to visit the Department of Mathematics at the University of Texas at Austin. In 1973 the mathematics department persuaded Bing to accept a permanent position at UT. Bing believed that part of the fun of life was to take on a variety of challenges. When he accepted the position at UT, he came with the idea of building the mathematics department into one of the top 10 state university mathematics departments in the country. While he was at Texas from 1973 until his death in 1986, he helped to improve the research standing of the department by recruiting new faculty and by helping to change the attitudes and orientation of the existing faculty. Raising research standards was the watchword of that period and is the guiding principle for the mathematics department now. Bing was chairman of the department from 1975 to 1977, but he used his international prominence for recruiting purposes throughout his stay at UT. The Department of Mathematics was considered one of the most improved departments over the period of Bing's tenure. The 1983 report of the Conference Board of Associated Research Councils listed Texas as the second most improved mathematics department in research standing during the period 1977-82, ranking it number 14 among state university mathematics departments at that time. The strategy of research improvement has continued in the UT

Department of Mathematics through the present day, and Bing would certainly be proud to see the department's continued improvement in its research stature.

Bing accomplished much during his life and left us with many ideas, personal and mathematical, to consider and enjoy. He left topologists a treasure-trove of theorems and techniques and left the UT Department of Mathematics with a goal and 13 years of good progress toward it. He was a man of strong character and integrity who liked to understand things for himself. For example, he never claimed to understand a theorem unless he personally knew a proof of it. He made decisions based on his own experience, relying on his independent judgment of a person or a cause whenever possible rather than averaging the opinions of others. He was a kind man and respected people for their own merits rather than measuring them on a single scale.

R. H. Bing died on April 28, 1986. He suffered from cancer and heart trouble during his last years, but he never complained about his health problems nor did he allow discomfort to dampen his enthusiasm and good spirits. He was an exemplary person. His friends, his family, his students, and the mathematical community have been enriched beyond bound by his character, his wisdom, and his unfailing good cheer, and continue to be enriched by his memory.

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